Week 1, 2 – Algebra of Matrices

(1)If $A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 3 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 3 \\ 4 & 1 \end{bmatrix}$

Find A + B, A + B + C, A + 2B + 3C, A`, C`, AB, AC, ABC, |A|, |B|, |AB|, |BA|.

(2)If $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 8 \\ -2 & 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 5 & 6 \\ 0 & 4 & 1 \end{bmatrix}$

Find A + B, AB, BA, |A|, |B|, |A + B|, |A| + |B|, |AB|, |BA|, |A|.|B|

(3)If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 3 \\ 1 & 2 \\ 3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 4 \\ -1 & 0 & 2 \end{bmatrix}$

Find if possible : A + B, A + B^t , A.B , A + C , A.C , B.C , |A| , |C| , |A.B|

(4)Find the inverse of each matrix, if exists :

$A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 & 2 & -1 \\ 2 & 3 & 1 & 2 \end{bmatrix}$

$E = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 5 & 6 \\ 0 & 4 & 1 \end{bmatrix}$, $F = \begin{bmatrix} -2 & 2 & 1 \\ 1 & 3 & 6 \\ 0 & 1 & 1 \end{bmatrix}$, $G = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 0 & 1 & 1 \end{bmatrix}$

Week 3 – Eigenvalues and Eigenvectors

(1)Find the eigenvalues and eigenvectors of the following matrices, if possible:

$A = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -2 & -2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$

(2)Find the eigenvalues and eigenvectors of the inverse of the matrix A given in (1).

Show the relation between the eigenvalues of A and its inverse.

(3)Find the eigenvalues and eigenvectors of the following matrices, if possible:

$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

Week 4 – Linear Systems

(1)Write the augmented matrix of each of the following linear systems and determine the type of solution.

(a) $x + y = 5$ $2x - y = 1$	(b) $x + y = 5$ $2x - y = 1$ $-2x - 2y = -10$	(c) $x + y = 5$ $2x - y = 1$ $x + 2y = 8$	(d) $x + y - z = 4$ $2x - y + 3z = 5$ $-x - y + z = -4$
(e) $x + y = 4$ $y - x = 3$	(f) $x + 2y = 4$ $2x - y = 0$ $3x + y = -1$	(g) $x + y = 3$ $2x - z = 4$ $2y + z = 8$	(h) $x + 2y - z = 1$ $x - y + 3z = 3$ $2x + y + 2z = 5$

(2)Solve the following linear systems by Gauss, Crammer, inverse methods, if possible:

(a) $x + y = 3$ $3x - y = 1$	(b) $2x + y = 6$ $3x - y = 4$	(c) $x + y + z = 5$ $2x - y + z = 2$ $2x + 2y - z = 4$
(d) $x + y - z = 3$ $x - y + 2z = 5$ $2x + 2y - 2z = 6$	(e) $2x + y + 2z = 8$ $x - y + z = 1$ $x + y + 2z = 7$	(g) $x_1 + x_2 - x_3 + x_4 = 4$ $2x_1 + x_2 + x_3 - x_4 = 3$ $x_1 - x_2 + 2x_3 + x_4 = 6$ $-x_1 + x_2 + x_3 - x_4 = 0$

Week 5 – Finite Series and Binomial Expansion

(1)Expand each of the following :

(a) $\sqrt{1+3x}$ (b) $\sqrt{4-3x}$ (c) $\sqrt[3]{1-2x^2}$ (d) $\dfrac{1}{\sqrt{4-2x}}$ (e) $\dfrac{1}{x^2-4x+3}$

(2)Find the nth sum $\sum_{r=1}^{n} r(r+2)$

(3)Find the nth sum $\sum_{r=1}^{n} (r+2)(2r+1)$

(4)Find the nth sum $\sum_{r=1}^{n} r(r^2+2)$

(5)Find the nth sum $\sum_{r=1}^{n} r^2(2r+3)$

(6)Find the nth sum $\sum_{r=1}^{n} \dfrac{1}{(r+2)(r+3)}$

(7)Find the nth sum $\sum_{r=1}^{n} \dfrac{2}{(r+4)(r+5)}$

(8)Find the nth sum $\sum_{r=1}^{n} \frac{4}{r(r+1)(r+2)}$ (9)Find the nth sum $\sum_{r=1}^{n} \frac{9}{(2r-1)(2r+3)}$

(10)Find the sum of the first 20 terms from the series $\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+....$

Week 6 – Mathematical Induction

Prove that the following relations by mathematical induction :

(1) $\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+...+\frac{1}{(n+1)(n+2)}=\frac{n}{2(n+2)}$

(2) $\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+...+\frac{1}{(n+1)(n+2)}=\frac{n}{2(n+2)}$

(3) $\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+...+\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$

(4)$1+3+5+...+(2n-1)=n^2$

(5) $1^3+2^3+3^3+...+n^3=\frac{1}{4}[n(n+1)]^2$

(6) $1+\frac{1}{2}+\frac{1}{4}+...+\frac{1}{2^{n-1}}=2-\frac{1}{2^{n-1}}$

(7) $4+14+30+...+(3n^2+n)=n(n+1)^2$

(8) $3+6+18+...+2.3^{n-1}=3^n-1$

(9) $n^4+2n^3+n^2$ is divisible by 4.

(10) x^n-y^n is divisible by $x-y$, where x, y are integer numbers and $x \neq y$.